

# Nonlinear elongation of two-dimensional structures in electron temperature gradient driven turbulence

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The nonlinear evolution of the electron temperature gradient (ETG) driven mode can be described with a simple, two-dimensional reduced fluid model, similar to that used for the thermal Rossby wave system. Consistent with ballooning mode structure, primary instability drive with a strong anisotropy in wave number (i.e.,  $k_y \gg k_x$ ) is considered for the inviscid limit of the ETG model. The amplitude equation, describing the initial envelope modulations of this system, is derived using reductive perturbation methods. The dynamics of the intensity field variance in radial and poloidal directions, i.e., the two diagonal elements of the covariance tensor, which follow from the amplitude equation, are investigated in an attempt to determine the basins of attraction for forming zonal flow and streamer secondary structures. It is found that the focusing (or diffracting) effect of Reynolds stress is essentially stronger in the radial direction than it is in the poloidal direction. Further analysis of the structure in the radially elongated limit of the amplitude equation yields interesting results, such as a poloidally localized sheared soliton solution. The approach used here is broadly applicable. © 2004 American Institute of Physics. [DOI: 10.1063/1.1786941]

## I. INTRODUCTION

Electron thermal transport remains an enduring enigma to researchers in fusion plasma physics. In particular, a class of phenomena, such as the penetration of electron thermal transport through barriers where ion and particle transport are quenched, suggests that the electron transport is mediated by smaller scale fluctuations than the “usual suspects,” which are the drift-ITG (ion temperature gradient) modes. Such smaller scale fluctuations are intrinsically less susceptible to quenching by  $\mathbf{E} \times \mathbf{B}$  shearing. This state of affairs thus nurtured the growth of interest in electron temperature gradient driven turbulence, produced by the electron temperature gradient driven (ETG) modes.<sup>1–3</sup> ETG modes are similar to the ion temperature gradient driven modes, with the roles of electrons and ions reversed, since  $k_{\perp} \rho_i \gg 1$ . An important *difference* between these two models is the Boltzmann ion response for both waves and zonal flows. This is in contrast to the ITG modes, for which the electron response to zonal flow perturbations is hydrodynamic.<sup>4</sup> This disparity is due to the fact that the ion response is Boltzmann for *all*  $k_{\perp} \rho_i \gg 1$  perturbations, so that the zonal flows for the ETG mode also satisfy this condition (see, for example, Ref. 5 for a consistent treatment taking into account the effect of ITG shear flows on the electron dynamics). While ETG modes thus became an attractive candidate for the cause of electron thermal transport, it was not long before the problem of space-time eddy scale became paramount for the ETG model. In particular, the spatial scale of ETG modes ( $\Delta r \sim \rho_e$ ) is so small that “electron gyro-Bohm” transport ( $D \sim \rho_e v_{Te} / L_{\perp} \sim \sqrt{m_e / m_i} D_{GB}$ ) is too feeble to be relevant. This naturally sent researchers scurrying to identify means of coupling ETG excitation to larger scales. One such means is via “in-

verse cascade” to the electron skin depth scale  $c / \omega_{pe}$ . However, the resulting Kadomtsev–Ohkawa type transport coefficient is still too small to be of much interest. The alternative means is via the formation of radially extended, poloidally localized structures or eddies, referred to as streamers. Such structures could significantly boost the ETG transport well beyond the electron gyro-Bohm level, by increasing the effective step size. Considerable controversy circulates about the streamer concept. One simulation group (using a flux tube code) has observed large streamers and claimed an enhancement of  $\chi_e$  of about 50 times the electron gyro-Bohm estimate.<sup>6</sup> A second group (using a global code) reports weakly anisotropic streamers, with only a slight enhancement of transport.<sup>7</sup> A third group (also using a global code) recently reported the “sighting” of streamers but noted only a weak enhancement in transport.<sup>8</sup> It should be apparent that a serious effort to understand the dynamics of streamer formation is necessary to properly sort out these conflicting simulation results, upon which the viability of the ETG mode depends.

A second motivation for this work is the more theoretical question of pattern selection by secondary instabilities. In particular, it is widely thought that zonal flows are generated by modulational instability of an ensemble or “gas” of drift waves. However it is easy to show that secondary streamer instabilities may also occur. This then raises the questions of which secondary structure configuration the system will actually select, how the streamers and zonal flows will interact etc. Attempts to address such issues have been limited in scope to comparisons of modulational instability growth rates. Moreover, the situation is further complicated by the fact that the streamers may consist of, or at least be strongly “seeded” by, the residues of linear ballooning mode structures. Thus, it is only natural to ask the question of which type of secondary structure (zonal flow or streamer) is the

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natural “attractor” or “selected pattern” for the modulational growth dynamics. Note that this is equivalent of asking about the shape of a self-focusing “spot” (see Refs. 9 and 10) in an anisotropic medium. Hence, we address this question of pattern selection by determination of the basins of attraction for “collapse” of an initial state to a streamer or zonal flow secondary structures. Since this is a subtle question, which deeply probes the nonlinear physics of ETG modes, our investigation is conducted using an extremely simple two-fluid model for the evolution of the stream function and the electron pressure in planar geometry. Relevant confinement-related effects such as magnetic shear, linear coupling of poloidal harmonics, and kinetic refinements are neglected in the hope of isolating the basic trends in the strongly nonlinear dynamics by first constituting an anisotropic envelope equation of the generalized DNLS (derivative nonlinear Schrödinger) variety. Since previous work demonstrated that small scale anisotropy can impact the growth of larger scale structures, we consider an initial state of small scale eddies elongated in the  $\hat{x}$  direction. These loosely correspond to the linear ballooning mode structure. As a consequence, we average over modal scales only in the  $\hat{y}$  direction and assume no separation of scales between the mode and the envelope in the  $\hat{x}$  direction. Notice that this constitutes a qualitative improvement beyond those provided by previous analyses. We employ only a minimal model of dissipation via local diffusion, i.e., viscosity and thermal diffusivity) and then take the inviscid limit for the dynamics of the envelope, in order to simplify the calculations. As we are concerned primarily with identifying the shape of the attractor for modulations, rather than the details of modulational instability saturation, this should be sufficient. Having derived the envelope equation we consider the time evolution of normalized  $x$  and  $y$  variances  $\langle(\Delta x)^2\rangle$  and  $\langle(\Delta y)^2\rangle$ , and derive equations for the evolution of each. A more rapid decrease in  $\langle(\Delta x)^2\rangle$  than  $\langle(\Delta y)^2\rangle$  with time is indicative of collapse to a zonally structured attractor, while the reverse suggests that the streamers are the preferred structure. The aim here is to determine which self-focusing or shear amplification process (i.e., zonal flow or streamer formation) is the more robust, nonlinearly dominant one. Our results indicate that  $\langle(\Delta x)^2\rangle$  is “pulled” to zero faster than  $\langle(\Delta y)^2\rangle$ , so that *zonal flows are the preferred secondary structure*. This suggests that streamer formation is not a “natural tendency” of ETG dynamics, so that streamers that are observed in simulations are either remnants of the initial, linear evolution or more intermittent in nature. The implications of this result are discussed more detail in the conclusion. We note that while numerous analyses of secondary instabilities, convective cells, etc., exist in the literature, none of these address the secondary pattern selection problem, except via straightforward comparisons of modulational growth rates for different initial anisotropies. Thus, we believe this approach to the pattern selection problem to be conceptually different. We also note that given the isomorphism between the ETG and thermal Rossby systems discussed previously, our results may be of interest in a broader context.

The remainder of the paper is organized as follows. In

Sec. II we briefly introduce the model and describe the mean field approach that is used. In Sec. III, the dynamics of isotropic initial state are considered, and the corresponding amplitude equation is derived. This leads to a description of the evolution of one-dimensional of (1D) average field variances. Further evolution of modulations in the poloidally and radially elongated limits is considered in Secs. IV A and IV B, respectively. In Sec. IV B we also describe a sheared soliton solution for the radially elongated, poloidally localized case. Section V contains a discussion and conclusion.

## II. BASIC MODEL AND EQUATIONS

The minimal model that describes curvature driven ETG turbulence, is a set of reduced fluid equations, which employ viscosity, and thermal diffusivity as model dissipation coefficients, i.e.,

$$(\partial_t + \hat{z} \times \nabla \Phi \cdot \nabla)(1 - \nabla^2)\Phi + \partial_y(\Phi + P) + \nu \nabla^4 \Phi = 0, \quad (1)$$

$$(\partial_t + \hat{z} \times \nabla \Phi \cdot \nabla)P - \chi \nabla^2 P - r \partial_y \Phi = 0. \quad (2)$$

Here, a form of dimensionless drift wave variables is used, and

$$\begin{aligned} \Phi &\rightarrow \frac{e\phi}{T_i \epsilon_{*i}}, & P &\rightarrow \frac{\epsilon_R P_{e1}}{\epsilon_{*j}^2 P_{i0}}, & t &\rightarrow \Omega_e \epsilon_{*i} t, & \mathbf{x} &\rightarrow \mathbf{x} / \sqrt{\tau_e \rho_e}, \\ \epsilon_{*i} &= \frac{\sqrt{\tau_e \rho_e}}{L_n} = \frac{v_{*i}}{c_{*e}}, & \epsilon_{*e} &= -\frac{\sqrt{\tau_e \rho_e}}{L_{p*}} = \frac{v_{*e}}{c_{se}}, \\ \epsilon_B &= \frac{\sqrt{\tau_e \rho_e}}{L_B} = \frac{v_B}{c_{se}}, & r &= \frac{e_B \epsilon_{*e}}{\epsilon_{*i}^2}, \end{aligned}$$

where  $\tau_e = T_i / T_e$ . Notice also that by replacing  $\epsilon_{*i} \rightarrow \epsilon_{*i}(1 - \epsilon_B / \epsilon_{*i})$ , one may take into account the curvature correction to the electrostatic potential without changing the form of the model.

This is a simplified model, with constant local background gradients and no magnetic fluctuations (see Ref. 11 for a discussion of the effects of electromagnetic streamers). Toroidal effects are modeled by a constant curvature drift  $v_B$ , and  $v_{*i}$  and  $v_{*e}$  are the ion and the electron diamagnetic drifts. Of course, for electromagnetic ETG further corrections enter at the skin depth scale  $c / \omega_{pe}$ . Notice also that the ion response is assumed adiabatic in the derivation of this model. This is valid here since  $k_{\perp} \rho_i \gg 1$  for both the waves and the zonal flows.

There is almost a one-to-one correspondence (see, for instance, Ref. 12) between the model, used here, and the thermal Rossby wave model, which is usually employed in studies of fluids in rapidly rotating systems<sup>13</sup> such as the Jovian–Venusian atmospheres and the jet stream. The main difference of this mode from that used by Busse and his co-workers (see, for example, Ref. 14) is the small divergence term, which couples the equations of density and vorticity, i.e., the dominant time dependence here is  $\partial_t \Phi$  instead of  $\partial_t \nabla^2 \Phi$ . The combined effects of quasi-neutrality and the Boltzmann ion response effectively define the potential vor-

ticity for the ETG mode to be  $\Phi - \tau_e \rho_e^2 \nabla^2 \Phi$ . This is in agreement with the Charney model of atmospheric Rossby waves.<sup>15</sup> Moreover, only the inviscid dynamics of the above model will be considered here. As stated above, we are concerned with identifying the flow accompanying the attractor for nonlinear modulations and not where it saturates. It is expected that the final saturation level *is* related to viscosity, but the initial trend of the modulations to be squeezed in one direction *is not*.

### A. Conservation laws

The inviscid limit of the ETG model is a Hamiltonian system. It has two conserved quantities:<sup>16</sup>

$$I_1 = \int \left( \Phi^2 + \nabla \Phi^2 - \frac{P^2}{r} \right) dV,$$

$$I_2 = \int \left( \left[ \Phi - \nabla^2 \Phi + \frac{P}{r} \right]^2 - [\Phi - \nabla^2 \Phi]^2 \right) dV.$$

The first one corresponds to the total energy of the system. When the mode is unstable, both fields ( $\Phi$  and  $P$ ) grow in such a way that the integral  $I_1$  remains constant, thanks to the minus sign in front of the  $P^2$  term. The second conserved quantity  $I_2$  is related to the enstrophy for this system. Indeed, one can think of it as an extension of enstrophy, which arises from pressure dynamics. It is the difference between the ‘‘generalized enstrophy,’’ calculated with the pressure field and the usual Hasegawa–Mima enstrophy, calculated without the pressure field. *Notice that it is not the generalized enstrophy, but the ‘‘differential enstrophy’’ that is conserved. This follows from the fact that as  $P \rightarrow 0$ ,  $I_2$  also vanishes.*

### B. Mean flow equations

To understand the large scale dynamics of the inviscid limit of the ETG equations from an envelope perspective (after Refs. 4 and 17), we average the motion over the rapid

oscillations of the primary mode. Here the length scale  $\Delta \mathbf{x}$  over which the average is taken corresponds to large dynamical scales, which are of order  $\epsilon^{1/2} \mathbf{x}$ , where  $\mathbf{x}$  is already normalized to  $\rho_{es} \equiv \rho_e \sqrt{\tau_e}$ . If we take  $\epsilon_{*i}$  as an estimate for the expansion parameter  $\epsilon$ , then the simple length scale for these averaged fields is  $\Delta \mathbf{X} \propto (\rho_{es}/L_n)^{1/2} (\mathbf{x}/\rho_{es}) \propto \mathbf{x}/\sqrt{\rho_{es} L_n}$ , i.e., the geometric mean of the electron Larmor radius and the background gradient scales. Field fluctuations, on the other hand, can be described in terms of the deviations from the mean value, which averages to zero,

$$\tilde{\Phi}(\mathbf{x}, t) = \Phi(\mathbf{x}, t) - \bar{\Phi}(\mathbf{X}, T).$$

Averaging the inviscid limits of Eqs. (1) and (2) we get the mean flow equations

$$\partial_T (1 - \nabla^2) \bar{\Phi} + \partial_Y (\bar{\Phi} + \bar{P}) = \overline{\hat{\mathbf{z}} \times \nabla \tilde{\Phi} \cdot \nabla \nabla^2 \tilde{\Phi}} = \{ \Phi, \nabla^2 \Phi \}, \quad (3)$$

$$\partial_T \bar{P} - r \partial_Y \bar{\Phi} = - \overline{\hat{\mathbf{z}} \times \nabla \tilde{\Phi} \cdot \nabla \tilde{P}} = \{ P, \Phi \}. \quad (4)$$

This means we implicitly assume that the mean field is described by the same model equations as the fluctuations, but is driven by the Reynolds stress and thermal flux generated due to the small scale dynamics. This follows from the assumption of adiabatic ion response for the flow as well as fluctuations, which is valid for  $k_{\perp} \rho_i \gg 1$ , as in the case of the ETG mode. This is in contrast to ITG, where the zonal flow and mode potentials respond qualitatively differently (i.e., hydrodynamically and adiabatically, respectively).

### C. Fluctuation dynamics

Subtracting the mean flow equations (3) and (4) from the original model equations (1) and (2), one obtains the equations for the fluctuations. It is possible to combine these into a single equation if higher order derivatives of mean fields are neglected, consistent with the perturbation expansion:

$$\underbrace{\{ \partial_t [\partial_t (1 - \nabla^2) + \partial_y] + r \partial_{yy} \}}_{L(\partial_t, \partial_y, \partial_x)} \tilde{\Phi} - \underbrace{(\hat{\mathbf{z}} \times \mathbf{k} \cdot \nabla) \{ [\omega (1 + 2k_y^2) - rk_y] \tilde{\Phi} - k_y \tilde{P} \}}_{N(\tilde{\Phi}, \tilde{P})} \tilde{\Phi} = 0. \quad (5)$$

Here  $L(\partial_t, \partial_y, \partial_x)$  is the linear operator for which the ETG mode is the unstable eigen function.  $N(\tilde{\Phi}, \tilde{P})$  represents advection by the mean flow. Modulations of quasimonochromatic wavelike fluctuations,

$$\tilde{\Phi}(\mathbf{x}, t) = \tilde{\Phi}_0(\mathbf{x}, t) + \epsilon \tilde{\Phi}_1(\mathbf{x}, t) + \dots, \quad (6)$$

where

$$\tilde{\Phi}_n(\mathbf{x}, t) = \Phi_n(\mathbf{X}, \tau, T_1) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} + \text{c.c.} \quad (7)$$

and similarly for  $P$ , are considered. Here the fluctuations are written with the assumption of scale separation between envelope modulations and the primary ETG dynamics, with  $\mathbf{X}, T$ , and  $\tau$  representing slow dynamical scales.<sup>18</sup> The expansion (6), the actual form of which depends on the anisotropy of the modulations, corresponds to a small amplitude expansion in  $e\phi/T_i$  starting with a term of order  $\epsilon_{*i}$  (since  $e\phi/T_i = \epsilon_{*i} \Phi$ ).

In Eq. (7)  $\omega$  and  $\mathbf{k}$  are related by the inviscid limit dispersion relation, which results from action of  $L$  on  $\tilde{\Phi}$ , i.e.,

$$\omega^2(1+k^2) - \omega k_y + r k_y^2 = 0$$

and has two roots,

$$\omega_{\pm} = \frac{k_y}{2(1+k^2)} [1 \pm \sqrt{1-4r(1+k^2)}], \quad (8)$$

which correspond to the phenomena of drift wave and convection cell. Notice that as  $r$  approaches zero, i.e., no ETG drive), the  $\omega_+$  branch becomes equal to the drift wave frequency and the  $\omega_-$  branch vanishes. Above the marginal stability boundary the  $\omega_+$  branch becomes unstable and the  $\omega_-$  branch decays.

#### D. Reynolds stresses

The Reynolds stress acting on the mean field generated by fluctuations of the form (7) can be expressed as

$$\overline{\{\tilde{\Phi}, \nabla^2 \tilde{\Phi}\}} = \nabla \cdot \nabla : \overline{(\nabla \tilde{\Phi} \hat{\mathbf{z}} \times \nabla \tilde{\Phi})} \approx \nabla \cdot \nabla : (2\mathbf{k} \hat{\mathbf{z}} \times \mathbf{k} |\Phi|^2 + \mathbf{k} \hat{\mathbf{z}} \times \mathbf{J} + \mathbf{J} \hat{\mathbf{z}} \times \mathbf{k}) \quad (9)$$

or

$$\overline{\{\tilde{\Phi}, \nabla^2 \tilde{\Phi}\}} \approx 2(\hat{\mathbf{z}} \times \mathbf{k} \cdot \nabla)(\mathbf{k} \cdot \nabla) |\Phi|^2 + (\hat{\mathbf{z}} \times \mathbf{k} \cdot \nabla)(\nabla \cdot \mathbf{J}) - (\mathbf{k} \cdot \nabla)(\nabla \times \mathbf{J})_z, \quad (10)$$

where

$$\mathbf{J} = i(\Phi \nabla \Phi^* - \Phi^* \nabla \Phi)$$

is the Schrödinger intensity flux density and  $\Phi$  is the complex amplitude. Similarly for the pressure advection nonlinearity,

$$\overline{\{\tilde{P}, \tilde{\Phi}\}} = \frac{1}{2} \nabla \cdot \overline{(\tilde{\Phi} \hat{\mathbf{z}} \times \nabla \tilde{P} - \tilde{P} \hat{\mathbf{z}} \times \nabla \tilde{\Phi})} \approx \nabla \cdot (\hat{\mathbf{z}} \times \mathbf{k} [\Phi^* P - P^* \Phi] + \hat{\mathbf{z}} \times [\Phi^* \nabla P - P^* \nabla \Phi]).$$

The phase difference between  $\tilde{\Phi}$  and  $P$  can be calculated from Eq. (2) by including slow modulations.

### III. EVOLUTION OF INITIALLY ISOTROPIC MODULATIONS

To understand the formation of elongated structures, we first need to understand the evolutionary dynamics<sup>18-20</sup> of the isotropic modulations [i.e.,  $\partial_X \sim O(\epsilon^{1/2})$ ,  $\partial_Y \sim O(\epsilon^{1/2})$ ,  $k_x \sim O(\epsilon^{1/2})$ , and  $k_y \sim O(1)$ ] of the fluctuation amplitude. For the isotropic limit, as there is no separation between  $k_x$  and  $\partial_X$ , it is necessary to take the *exact*  $x$  dynamics into account.

This can be carried out by calculating the average Reynolds stress drives taking the averages in the  $y$  direction only. This is achieved by writing

$$\tilde{\Phi}_n(\mathbf{x}, t) = \Phi_n(x, Y, \tau, T) e^{i(k_y y - \omega t)} + \text{c.c.}, \quad (11)$$

where the  $x$  dependence of the field is kept exactly, but scale separation in the  $y$  direction is assumed. If one calculates the Reynolds stresses using this procedure, the dominant contribution turns out to be the same as the  $k_y \gg k_x$  limit of the general Reynolds stress formula (10), since in this limit the dominant contribution (i.e.,  $2k_y^2 \partial_X \partial_Y |\Phi|^2$ ) is independent of  $k_x$ . For either a stable or a weakly unstable mode, pressure advection may easily be calculated as

$$\{P, \Phi\} = \frac{r k_y^2}{\omega^2} \partial_X \left( \partial_{T_1} + \frac{\omega}{k_y} \partial_Y \right) |\Phi|^2. \quad (12)$$

These expressions for the Reynolds stress and thermal flux yield the mean field equations

$$\partial_T \bar{\Phi} + \partial_Y (\bar{\Phi} + \bar{P}) = -2k_y^2 \partial_{XY} |\Phi|^2, \quad (13)$$

$$\partial_T \bar{P} - r \partial_Y \bar{\Phi} = \frac{r k_y^2}{\omega^2} \partial_X \left( \partial_{T_1} + \frac{\omega}{k_y} \partial_Y \right) |\Phi|^2 \quad (14)$$

and the equations for the evolution of the fluctuation amplitude via the action of these mean fields.

We take  $\bar{\Phi} \sim O(\epsilon^{1/2})$  to balance the fluctuation drive with the mean field for the (possibly) unstable  $\omega_+$  mode:

$$\underbrace{\{\partial_t [\partial_t (1 - \nabla^2) + \partial_y] + r \partial_{yy}\}}_{L(\partial_t, \partial_y, \partial_x)} \Phi + \underbrace{k_y \partial_X \{[\omega(1 + 2k_y^2) - k_y] \bar{\Phi} - k_y \bar{P}\}}_{N(\bar{\Phi}, \bar{P})} \Phi = 0. \quad (15)$$

We then perform a modulational stability analysis, by substituting Eq. (11) into Eq. (15), and assume scale separation between various time scales, i.e.,

$$t \rightarrow t_f + T_1 \epsilon^{-1/2} + \tau \epsilon^{-1} \Rightarrow \partial_t \rightarrow -i\omega + \epsilon^{1/2} \partial_{T_1} + \epsilon \partial_\tau,$$

$$y \rightarrow y_f + \epsilon^{-1/2} Y \Rightarrow \partial_y \rightarrow ik_y + \epsilon^{1/2} \partial_Y,$$

where the subscript  $f$  denotes the fast time scale corresponding to the rapid phase motion of the wave inside the en-

velope. In practice, this expansion corresponds to a Taylor series expansion of the operator  $L$ , with respect to its first two arguments, from the “point”  $(-i\omega, ik_y, \partial_X)$  to the neighboring point  $(-i\omega + \partial_{T_1} \epsilon + \partial_\tau \epsilon^2, ik_y + \partial_Y \epsilon^{1/2}, \partial_X)$ . We will also consistently neglect fourth and higher order radial dispersion terms.

This expansion scheme yields the linear dispersion relation to zeroth order. The first order “secular” terms represent the group motion of the envelope and can be eliminated by transforming to a frame moving with the group velocity. No-

tice that the group velocity, as it is defined here, is strictly in the  $y$  direction. This is because of the ordering that is used (i.e.,  $k_y \gg k_x$ ) which forces the group motion effects in the  $x$  direction to be of higher order. Furthermore, we do not separate the  $x$  dynamics into large and small scales for this particular case. Hence, such effects are already retained *exactly* in the equations. Thus, for example, a zonal flow with “absolute growth” (i.e.,  $\gamma > v_{gx}$ ) is encompassed within this approach (since  $v_{gy} > \gamma > v_{gx}$  is easily possible). However, growth rates faster than the poloidal group motion of the primary mode do not correspond to modulational instability in the classical sense. This is because in the case of such rapid growth, if the average dynamics is to be relevant, it has to be as fast as the fluctuation dynamics, which means that it cannot be regarded as a modulation anymore. In this case, the direct three- or four-wave coupling picture is far more convenient. It should be noted, however, that it is quite possible that the linearly growing mode, which is growing as a result of a direct wave coupling process, actually grows more slowly than it travels in the poloidal direction. Then, one can argue that this mode should also be spatially modulated, in addition to the direct wave coupling. In any case, we assume that  $v_{gy} > \gamma$  for the primary mode that is modulated and pass to a frame moving in the  $y$  direction by setting  $\partial_{T_1} = -v_{gy}\partial_Y$ . The equations for the mean fields [i.e., Eqs. (13) and (14)] can then be solved to yield

$$\bar{\Phi} = - \left( \frac{(r/\omega^2)(v_{gy} - \omega/k_y) + 2v_{gy}}{v_{gy}(1 - v_{gy}) - r} \right) k_y^2 \partial_X |\Phi|^2, \quad (16)$$

$$\bar{P} = - \left( \frac{(r/\omega^2)(1 - v_{gy})(v_{gy} - \omega/k_y) - 2r}{v_{gy}(1 - v_{gy}) - r} \right) k_y^2 \partial_X |\Phi|^2. \quad (17)$$

It should be noted at this point that if we chose the  $\omega_-$  branch, the scaling of  $\bar{\Phi}$  would be  $O(\epsilon^{1/2}/r)$ , and for stable modes  $r < 1/8$ . However, the amplitude of the  $\omega_-$  branch is

$$\alpha = \frac{[\omega(1 + 2k^2) - k_y] \left[ r \frac{1}{\omega^2} (v_g - v_\phi) + 2v_g \right] - r \left[ \frac{1}{\omega^2} (v_g - 1)(v_g - v_\phi) - 2 \right]}{[v_g(v_g - 1) + r][k_y - 2\omega(1 + k_y^2)]} k_y. \quad (20)$$

Notice that  $\alpha$  becomes particularly large near the marginal stability curve [i.e., as  $\omega \rightarrow k_y/2(1 + k^2)$ ], thus boosting the importance of the nonlinearity on the overall dynamics. Equation (18) is the amplitude equation governing the weakly nonlinear evolution of isotropic, 2D, ETG structures. It is, in a sense, a nonlinear dispersion relation, valid for both the  $\omega_+$  and the  $\omega_-$  branches (nonlinear coupling neglected).

To complete the analysis, we write the Lagrangian density for Eq. (18) as

not normally expected to be sufficient to drive modulational instability since it is a linearly decaying mode. Of course one should also consider the case when the damped mode is strongly driven by other (unstable) modes. Notice that in the initial phase of the three-wave coupling process (see, for example, Ref. 21), where  $\alpha$  the pump amplitude is large, the growth generated by the pump always wins over the decay rate, since the linear growth rates of the decaying mode and the mean flow are proportional to  $\Gamma = -\gamma/2 \pm (\gamma^2 + 4\alpha|\eta_k|)^{1/2}/2$ , where  $\alpha$  is the coupling (which should be positive for constructive coupling),  $\eta_k = \Phi_k + (\omega_-/rk_y)P_k$  is the amplitude of the growing “normal coordinate” of the pump wave, and  $-\gamma$  is the decay rate of the linearly decaying mode. In reality, a more comprehensive analysis of the modulation of both the growing and the damped modes that are nonlinearly coupled should be considered. Here we assume that the nonlinear coupling between the growing and decaying modes is small, so as to obtain a single amplitude equation in the end.

Substituting Eqs. (16) and (17) into Eq. (15), and then expanding and dividing by  $-[2(1 + k_y^2)\omega - k_y]$ , we obtain the amplitude equation

$$i\partial_\tau \bar{\Phi} + \frac{1}{2} \frac{\partial^2 \omega}{\partial k_y^2} \partial_{YY} \bar{\Phi} + \frac{1}{2} \frac{\partial^2 \omega}{\partial k_x^2} \partial_{XX} \bar{\Phi} + \alpha k_y^2 \partial_{XX} (|\bar{\Phi}|^2) \bar{\Phi} = 0 \quad (18)$$

with the definition

$$\frac{\partial^2 \omega}{\partial k_x^2} \equiv - \frac{2\omega^2}{[2(1 + k_y^2)\omega - k_y]}. \quad (19)$$

Here  $\alpha$  is

$$L = \frac{i}{2} (\Phi^* \partial_\tau \bar{\Phi} - \Phi \partial_\tau \Phi^*) - \beta_x |\partial_X \Phi|^2 - \beta_y |\partial_Y \Phi|^2 - \frac{\alpha}{2} k_y^2 [\partial_X (|\Phi|^2)]^2,$$

where

$$\beta_x \equiv \frac{1}{2} \frac{\partial^2 \omega}{\partial k_x^2}, \quad (21)$$

$$\beta_y \equiv \frac{1}{2} \frac{\partial^2 \omega}{\partial k_y^2}, \quad (22)$$

and  $\alpha$  is as already given in Eq. (20). It is also possible to rewrite the amplitude equation with the scalings  $X \rightarrow X/\sqrt{|\beta_x|}$ ,  $Y \rightarrow Y/\sqrt{|\beta_y|}$ ,  $\Phi \rightarrow (k_y \sqrt{|\alpha/\beta_x|})\Phi$ , and replacing  $\Phi \rightarrow \Phi^*$  if  $\beta_x < 0$ , yielding

$$i\partial_\tau \Phi + \partial_{YY} \Phi + \partial_{XX} \Phi + \sigma \partial_{XX} (|\Phi|^2) \Phi = 0. \quad (23)$$

Notice that this scaling uses the fact that the signs of  $\beta_x$  and  $\beta_y$  are the same and

$$\sigma = \text{sgn}[\alpha/\beta_x] = \pm 1.$$

We shall see that  $\sigma = -1$  corresponds to an attractive nonlinearity and  $\sigma = +1$  to a repulsive one. It should also be noted here that  $\beta_x/\beta_y \approx 2/3$ , so we do not scale one dimension more strongly than the other.

On account of the time independent Lagrangian density, the total Hamiltonian is conserved,

$$H = \int dV \left[ |\partial_X \Phi|^2 + |\partial_Y \Phi|^2 + \frac{\sigma}{2} [\partial_X (|\Phi|^2)]^2 \right]. \quad (24)$$

Equation (23) further implies conservation of intensity,

$$\partial_\tau |\Phi|^2 + \partial_X J_x + \partial_Y J_y = 0$$

and momentum density

$$\partial_\tau \mathbf{J} + \nabla \cdot \mathbf{S} = 0$$

with

$$S_{ij} = 2(\partial_i \Phi^* \partial_j \Phi + \partial_i \Phi \partial_j \Phi^*) + \sigma \delta_{ij} (\partial_X |\Phi|^2)^2 + 2\sigma \delta_{jX} \partial_i |\Phi|^2 \partial_X |\Phi|^2 - \sigma \delta_{ij} \partial_{XX} |\Phi|^4 - \partial_i \partial_j |\Phi|^2$$

being the effective radiation stress tensor. Notice that this system is analogous to an anisotropic fluid dynamics for which  $\rho = |\Phi|^2$  and  $v = 2\nabla \theta$ , for  $\Phi = |\Phi| \exp(i\theta)$ , with an unusual “equation of state” implied by the structure of the radiation stress tensor.

Our aim is to identify the final state of secondary elongation and anisotropy structure. Therefore the quantities

$$V_x = \langle X^2 \rangle = \int X^2 |\Phi|^2 dXdY,$$

$$V_y = \langle Y^2 \rangle = \int Y^2 |\Phi|^2 dXdY$$

are of particular interest. They represent the mean square widths of a wave packet in  $x$  and  $y$  directions, i.e.,  $V_x$  represent the  $Y$  average of the one-dimensional  $X$  variance of the field cross sections taken at different values of  $Y$ . For example, a distribution with  $V_x \gg V_y$  is radially elongated, and hence corresponds to the streamer limit. The important caveat with using these quantities is the fact that they give useful information only when the structure is elongated in either  $x$  or  $y$  direction but *not* in some arbitrary direction. In other words, these are the diagonal elements of the more general covariance tensor, and they make sense only when  $x$  and  $y$  are at least close to the principal axes for that tensor.

The equations that govern the evolution of the variances are easily shown to be

$$\frac{d^2 V_x}{dt^2} = 4H + 4 \int [\sigma (\partial_X |\Phi|^2)^2 + (|\partial_X \Phi|^2 - |\partial_Y \Phi|^2)] dXdY, \quad (25)$$

$$\frac{d^2 V_y}{dt^2} = 4H + 4 \int [|\partial_Y \Phi|^2 - |\partial_X \Phi|^2] dXdY. \quad (26)$$

Here  $H$  is the total Hamiltonian given in Eq. (24), which is a conserved quantity. This is the same as the corresponding dynamics for the 2D nonlinear Schrödinger equation NLS,<sup>22</sup> except for the first term in the integral in Eq. (25). These variance equations can thus be written in the form

$$\frac{d^2 V}{dt^2} = \frac{d^2 (V_x + V_y)}{dt^2} = 8H + 4 \int \sigma (\partial_X |\Phi|^2)^2 dXdY, \quad (27)$$

$$\frac{d^2 (V_x - V_y)}{dt^2} = 8H - 16 \int |\partial_Y \Phi|^2 dXdY. \quad (28)$$

As these are explicit equations for the second derivatives of the variances (akin to acceleration of the variances), one may qualitatively interpret them as Newton’s equations of motion. For  $H < 0$ , which is possible for  $\sigma = -1$ , the total variance  $V$  is always pulled towards zero. Thus the width of an initially stationary disturbance of the field becomes singular before  $t < t_c = \sqrt{V_0/4H}$ , where  $V_0$  is the initial variance. Here,  $t_c$  is the collapse time of the critical (i.e., 2D cubic) NLS system. This system collapses faster, due to the additional “pull” by the nonlinear stress in Eq. (27).

On the other hand,  $H < 0$  implies that the quantity  $V_x - V_y$  in Eq. (28) is always pulled towards the negative axis. This means  $V_x$  is always pulled more strongly than  $V_y$ . Since an initially isotropic state corresponds to  $V_x \sim V_y$ , one arrives at the conclusion that the final state will have  $V_x \ll V_y$ . Thus poloidally elongated zonal flow type eddies appear to be preferred.

This is indeed rather surprising, because it implies that the zonal flows are favored in ETG-type dynamics, given that the primary modes that drive the instability are radially extended and the state which is initially modulated is roughly isotropic. This suggests that the linear mode structure is not a good indicator of the anisotropy of the final state of the system. Notice that this method gives a general idea about the overall, statistically averaged structure of the field elongation. It is quite possible that the poloidally elongated envelope breaks up into small streamers which are later smoothed out by the averaging procedure, though.

Finally, looking at Eqs. (25) and (26), one can claim that the effect of nonlinearity is stronger on  $V_x$  than it is on  $V_y$ . So if the nonlinearity is repulsive (i.e.,  $\sigma = +1$ ) it pushes  $V_x$  more strongly. This makes  $V_y < V_x$ , ultimately. However, in this case, no shear layer is generated, and the resulting flow is relatively weak and dispersed. It is also possible to argue that nonlinear interaction between “self-focusing” and “self-diffracting” modes determines the final shape of the structure. An estimation of the importance of such an effect is still

required, and will be pursued in the future. Such an analysis requires building a model of a multicomponent, interacting soliton gas, which is indeed a formidable task and is left to the future.

#### IV. FURTHER ANISOTROPIC EVOLUTION

The amplitude equation used above for an anisotropically collapsing field will be valid only for a short time. After the initial time period, a corresponding anisotropic evolution equation should be used. Here, we first study further evolution in the poloidally elongated limit.

##### A. Zonal flow formation

An equation describing the further evolution of the structure in the limit  $\partial_X \gg \partial_Y$  may be derived in a manner similar to that used for the isotropic case. The main problem is that if the scaling is such that  $\partial_Y \sim \partial_X^2$ , one can no longer neglect the divergence of the intensity flux density  $k_y(\partial_X \nabla \cdot \mathbf{J})$  in the calculation of the Reynolds stress. It is possible that this term nonlinearly damps the zonal flow, thus balancing the self-focusing tendency. Such an effect does not appear in pure eikonal theory treatments. Thus, the critical scaling  $\partial_X \sim O(\epsilon^{1/4})$  corresponds to the scale at which the collapse may stop. Again using  $\epsilon_{*i}$  as a crude estimate for  $\epsilon$ , we get  $\Delta X \sim \rho_{es}^{3/4} L_n^{1/4}$  for the radial scale of the final mesoscale structure. If this structure is a zonal flow, i.e., no breakup into smaller scale structures occurs), then this scale would correspond to the radial size of the zonal flow. However, as mentioned before, it is also possible that breakup into smaller scale structures occurs. Then, this estimate would correspond to the radial size of these ‘‘small scale’’ instabilities, which is still  $\epsilon^{-1/4}$  times larger than the correlation length for the ETG mode, i.e.,  $\rho_{es}$ ).

Before the critical scaling regime, however, there is an intermediate regime. Since this corresponds to a poloidally collapsed state of an initially isotropic field, the scaling of  $\partial_Y$  should be kept as before [i.e.,  $\partial_Y \sim O(\epsilon^{1/2})$ ,  $\partial_X \sim O(\epsilon^{1/3})$ ,  $k_x \sim O(\epsilon^{1/2})$ , and  $k_y \sim O(1)$ ] [avoiding the critical scaling  $\partial_X \sim O(\epsilon^{1/4})$ ] in order to be able to neglect the divergence of the intensity flux density term. This, in turn, results in a scaling of the mean field as  $\bar{\Phi} \sim O(\epsilon^{1/3})$  and time as  $\partial_T \sim O(\epsilon^{1/2})$  and  $\partial_T \sim O(\epsilon^{2/3})$ . Then, the dynamics of fluctuations in a frame moving with velocity  $\mathbf{v} = v_{gy} \hat{\mathbf{y}}$  can be described with a simple amplitude equation as before, which is

$$i\partial_T \Phi + \beta_x \partial_{XX} \Phi + \alpha k_y^2 \partial_{XX} (|\Phi|^2) \Phi = 0,$$

where  $\alpha$  and  $\beta_x$  are still the same, and the scaling which allows us to neglect the dispersion in  $y$  direction makes the Hamiltonian even more negative. This equation is still attractive, and this fact suggests that the collapse will continue until the mean flow component accompanying the field forms a singular shear layer. However, the  $k_y \partial_X (\nabla \cdot \mathbf{J})$  term, if included, may act to reduce the mean flow, and therefore may balance this tendency.

##### B. Streamer formation

For the ETG mode, the possibility that the zonal flows are damped or balanced during the isotropic phase due to other physical processes<sup>23</sup> should also be considered. In this case  $V_x$  will remain fixed (or oscillate) after some initial collapse, while  $V_y$  would continue collapsing after the fluctuations reach a certain level. This leads us to the state in which the anisotropy of the drive and the modulations are the same, i.e.,  $k_x \partial_Y \sim k_y \partial_X$ . Choosing the scaling as  $\partial_X \sim O(\epsilon)$ ,  $\partial_Y \sim O(\epsilon^{1/2})$ ,  $k_x \sim O(\epsilon^{1/2})$ , and  $k_y \sim O(1)$ , and implementing a similar analysis as in the isotropic case, one can derive the amplitude equation:

$$i\partial_T \Phi + \frac{1}{2} \frac{\partial^2 \omega}{\partial k_y^2} \partial_{YY} \Phi - \frac{1}{2} \frac{\partial^2 \omega}{\partial k_x^2} k_x^2 \Phi + \alpha \Phi (\hat{\mathbf{z}} \times \mathbf{k} \cdot \nabla)^2 |\Phi|^2 = 0. \quad (29)$$

Notice that apart from resonance, i.e., considerable enhancement of  $\alpha$ , near marginal stability), the terms in this equation can only be balanced by choosing an expansion of the form  $\tilde{\Phi} = \epsilon^{-1/2} \tilde{\Phi}_0 + \epsilon^{1/2} \tilde{\Phi}_1 + \dots$ , which is a rather strong scaling of the field, producing  $e\Phi/T_i \sim O(\epsilon_{*i}/\epsilon^{1/2})$ . For this to be meaningful, it should be true that  $1 \gg \epsilon \gg \epsilon_{*i}^2$ . Physically, since we still neglect the ITG effects, our ‘‘large scale’’ structures are smaller than the ion Larmor radius. The above condition effectively requires the large scale structures to be smaller than the density gradient length scales. This should still be very well satisfied for the ETG system. Also, for  $\epsilon \sim \epsilon_{*i}$ , as a crude estimate for the expansion parameter, the fluctuation level is estimated to be  $e\Phi/T_e \sim O(\epsilon_{*i}^{1/2})$ .

Notice also that a three-scale expansion [ $\partial_X \sim O(\epsilon^{1/2} \delta^{1/2})$ ,  $\partial_Y \sim O(\delta^{1/2})$ ,  $k_x \sim O(\epsilon^{1/2})$ , and  $k_y \sim O(1)$ ], where  $\delta$  is a secondary expansion parameter such as  $\epsilon$  but larger in comparison (i.e.,  $\epsilon < \delta < 1$ ), can be used to study to describe the breakup of the zonal flow or the initially isotropic convective cell. This corresponds to the stage during the collapse where the initial scaling [i.e.,  $\partial_X \sim O(\epsilon^{1/2})$ ,  $\partial_Y \sim O(\epsilon^{1/2})$ ,  $k_x \sim O(\epsilon^{1/2})$ , and  $k_y \sim O(1)$ ] suddenly breaks down and structures with  $\partial_Y \sim O(\delta^{1/2})$  appear (a scale describing the poloidal modulations which can be seen in Fig. 1). It is not totally clear, however, what the appropriate inverse scale length for this second parameter actually is. This parameter defines the scale length onto which the former nonlinear structure breaks up. In general, we may speculate that whereas  $\epsilon$  is the ratio of  $\rho_{es}$  to the largest background gradient scale,  $\delta$  could be the ratio of  $\rho_{es}$  to the smallest one, i.e.,  $\epsilon_B$  and  $\epsilon_{*i}$  correspondingly) or even other scale lengths depending on the physics attributed to the breakup process and resulting from symmetry breaking.<sup>24,25</sup> In practice, this implies a perturbation expansion for the  $y$  direction (via the parameter  $\delta$ ) independent of the large scale evolution in the  $x$  direction (the parameter  $\epsilon$ ). This approach yields the same amplitude equation as Eq. (29), except the fact that the term proportional to  $k_x^2$  does not appear in the final equation (notice that this term can be transformed away by considering it as a shift to the linear ‘‘ground-state’’ frequency).

In any case, Eq. (16) can be thought of as a model amplitude equation that describes the evolution of streamer dy-

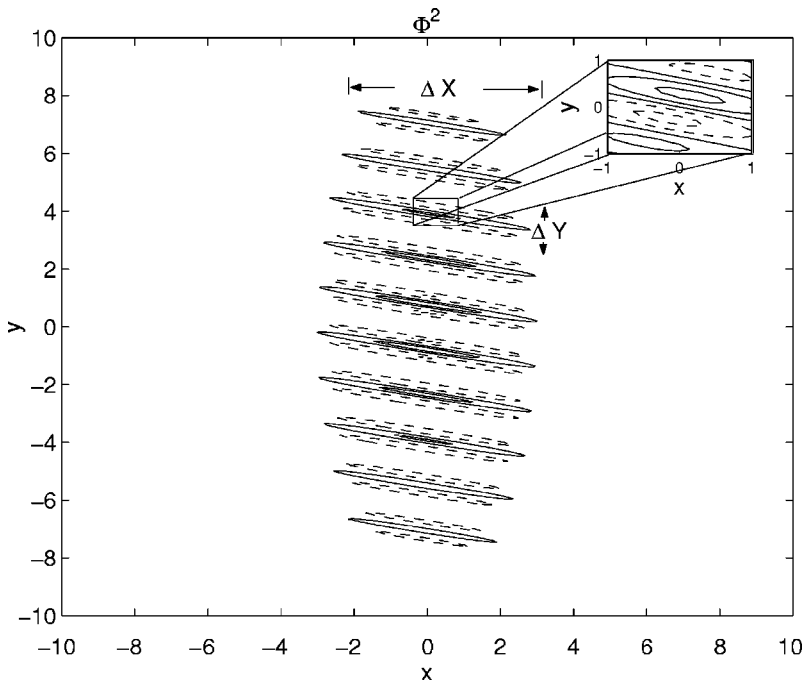


FIG. 1. Breakup of the zonal flow into radially elongated structures of intermediate scale between the small scales and the mesoscales. Still, an individual “streamer” can be treated as a modulation. As can be seen for the intermediate (i.e.,  $\delta$ ) scales the streamer is not localized in the radial direction. However, its radial extension (apart from the slight tilt that can be seen) is finalized when the mesoscale (i.e., scale) is also taken into account.

namics in a “locally” radially elongated limit. To see its implications, we will derive the variance dynamics and apply the same method that we used for the isotropic case to see if collapse results. We first note, however, that the third term in Eq. (29) is a simple shift the basic ground-state energy. Thus, it can be added to the frequency and eliminated, i.e.,

$$\psi(\mathbf{X}, \tau) = \Phi(\mathbf{X}, \tau) e^{-i(\beta_x k_x^2 \tau)}.$$

Then, if we perform the transformation

$$\eta = \left( Y + \frac{k_x X}{k_y} \right) / \sqrt{|\beta_y|},$$

$$\xi = X / \sqrt{|\beta_y|},$$

$$\Phi = (k_x \sqrt{|\alpha' \beta_y|}) \Phi,$$

we obtain the dimensionless form of the amplitude equation

$$i \partial_\tau \psi + \partial_{\eta\eta} \psi + \sigma \psi \partial_{\xi\xi} |\psi|^2 = 0 \tag{30}$$

(with  $\sigma = -1$  for the attractive case). The total Hamiltonian, which can be written using the transformed variables as

$$H = \int \left( |\partial_\eta \psi|^2 + \frac{\sigma}{2} (\partial_\xi |\psi|^2)^2 \right) d\eta d\xi,$$

is also conserved. Using conservation of wave quanta and field momentum in the  $\eta$  direction (i.e., poloidal direction) and assuming the field is localized in this direction, one can write the equations for the averaged one-dimensional variances as

$$\frac{dV_\xi}{dt} = 0 \Rightarrow V_\xi = \text{const},$$

$$\begin{aligned} \frac{d^2 V_\eta}{dt^2} = \frac{d^2 V}{dt^2} &= 2 \int S_{\eta\eta} d\eta d\xi = \int [8 |\partial_\eta \psi|^2 \\ &+ 2\sigma (\partial_\xi |\psi|^2)^2] d\eta d\xi = 8H - 2\sigma \int (\partial_\xi |\psi|^2)^2 d\eta d\xi. \end{aligned}$$

Notice that Eq. (30) does not have linear dispersion in the  $\xi$  direction, which implies that there is no intensity flux in the  $\xi$  direction. The relation for  $V_\xi$  follows immediately from this observation, even without reference to field localization in the  $\xi$  direction. This is particularly important for the case of zonal flow breakup, because the scales on which the field is varying in the radial direction are large compared to the scales on which the field is varying in the poloidal direction. Thus, the field may not appear localized in our local approximation (i.e.,  $\delta$  scaling instead of  $\epsilon$ ), which is aimed at resolving the poloidal dynamics (i.e., streamer limit). This is an important drawback, which makes the estimation of the radial size of the streamer very difficult. However, previous estimates for the isotropic structure (i.e.,  $\epsilon$  scaling) can be used as a guideline, since the radial dynamics are assumed to be the same in this limit. The equation for  $V_\eta$  looks very similar to the corresponding variance equation for the one-dimensional cubic NLS. This implies that for the attractive nonlinearity, an arbitrary initial distribution tends to be attracted to a “natural state,” rather than collapsing completely. This state is a fixed point of the variance equation that appears at  $H = (\sigma/4) \int (\partial_\xi |\psi|^2)^2 d\eta d\xi$ . If this is an attractor, the solution may be expected to oscillate around this equilibrium (as for a breather).

It is possible to construct an exact solution of this amplitude equation using the idea of separation of variables from partial differential equations theory. Writing



$$\psi = F(\eta, \tau)G(\xi)$$

and dividing every term in Eq. (30) by  $G(\xi)F(\eta, \tau)|F(\eta, \tau)|^2$  we obtain two equations

$$\partial_{\xi\xi}|G(\xi)|^2 = \frac{m}{\sigma}, \quad (31)$$

$$i\partial_{\tau}F(\eta, \tau) + \partial_{\eta\eta}F(\eta, \tau) + m|F(\eta, \tau)|^2F(\eta, \tau) = 0, \quad (32)$$

where  $m$  is used as a separation parameter. *Solutions of the radial part [i.e., Eq. (31)] (notice that  $\sigma = -1$  for the attractive case, which is relevant) are not localized, since the solution  $|G(\xi)|^2 = |G_0|^2 + (m/2\sigma)(\xi - \xi_0)^2$  is like a local approximation to the tip of a function, which may be localized on larger scales. However, as noted before, localization in the radial direction is not necessary here. Equation (32) is the one-dimensional cubic NLS. Its properties are quite well known and it has soliton and soliton train solutions.<sup>22,26,27</sup> If we write down the single soliton solution in the original variables,*

$$\begin{aligned} \Phi(X, Y, \tau) = & \left( \frac{|G_0|^2}{m} + \frac{X^2}{2\sigma\beta_y} \right)^{1/2} \frac{1}{k_x} \sqrt{\frac{2a}{|\alpha/\beta_y|}} \\ & \times \operatorname{sech} \left[ \sqrt{|\alpha/\beta_y|} \left( Y + \frac{k_x}{k_y} X \right) - \left( \frac{U^2}{4} - a \right) \tau \right] \\ & \times \exp \left[ \frac{U}{2\sqrt{\beta_y}} \left( Y + \frac{k_x}{k_y} X \right) \right] \\ & \times - \left( \frac{U^2}{4} - \alpha + \beta_x k_x^2 \right) \tau. \end{aligned} \quad (33)$$

We find [in Eq. (33)] a self-sheared soliton solution of Eq. (29), with  $a$  and  $U$  being measures of amplitude and velocity of the soliton in the  $\eta$  direction, respectively. The soliton train solution can also be written in the same way. The stability to perturbations along the  $\eta$  direction follows from the NLS. Although the perpendicular perturbations should also be considered, one may argue that they will (possibly) only change the perpendicular (i.e., radial) structure of the solution, which is not very well described in this approximation anyway. In practice this solution describes the direction of elongation of the streamer (i.e., perpendicular to the  $\eta$  direction rather than exactly radial). Since this is a sheared coordinate that arise from the action of the Reynolds stress (i.e., from  $\hat{\mathbf{z}} \times \mathbf{k} \cdot \nabla$ ), one can conclude that this is indeed a “self-sheared” nonlinear solution. In a polar plot ( $X \rightarrow r - r_0, Y \rightarrow r_0\theta$ ) the self-sheared nonlinear solution looks like that shown in Fig. 2.

## V. CONCLUSION

In this paper, we have studied the dynamics of anisotropic evolution and collapse of modulations to zonal flow and streamer structures in ETG thermal Rossby wave turbulence. The aim here is to determine under what circumstances each of these anisotropic structures is preferred and to calculate the basins of attraction for each. The principal results of this paper are the following.

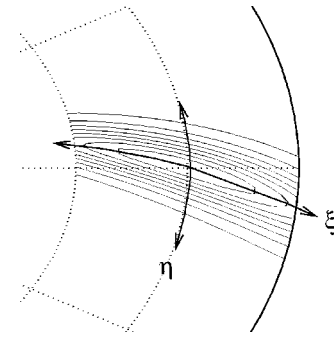


FIG. 2. Self-sheared soliton solution for the radially elongated limit amplitude equation given in Eq. (33).

- (1) The amplitude equations (for the mode intensity) corresponding to the radially and poloidally elongated limits of a secondary structure have been derived using reductive perturbation theory. Anisotropic primary eddy shape was presumed.
- (2) It is found that amplitude equations have a generalized DNLS structure and can have either attractive or repulsive nonlinearities. A Hamiltonian functional for the system, which can be negative, was identified.
- (3) There is an indication that collapse to radially narrow poloidally elongated mesoscale structures is preferred. Thus, the zonal flow structure is dynamically favored.
- (4) It is predicted that zonal-type structures continue to collapse until  $\partial_Y \sim \partial_X^2$ , at which point nonlinear damping becomes important. This corresponds to a zonal flow scale of  $\Delta X \sim \rho_{es}^{3/4} L_n^{1/4}$ , which sets the small scale cutoff of the ETG zonal flow spectrum.
- (5) A self-sheared nonlinear wave form with flow along almost radial slowly spiraling contours has been derived.

Of course, many simplifying approximations have been made, such as neglect of trapped-untrapped particle collisional friction and geometric details of the linear instability process. Thus the results, while quite basic and general, are also quite simplified. Nevertheless, our findings suggest the following:

- (1) Formation of zonal flow, rather than streamer, structure is favored.
- (2) The nonlinear collapse to form zonal shears selects a cutoff scale  $\Delta X \sim \rho_{es}^{3/4} L_n^{1/4}$  for the zonal flow spectrum.

These results, though quite theoretical in origin and character, have several interesting implications for ETG turbulence and electron thermal transport. First, they suggest that streamer formation is *not* a generic trend in the dynamics, so that the etiology of streamers is most likely as a remnant or residue of the linear ETG mode structure (i.e., the de-facto “initial conditions” for the nonlinear evolution), or as an artifact of the computation. Thus “streamer enhancement” of ETG transport is dubious, so that for modest plasma beta, electrostatic ETG is *not* a likely candidate for the explanation of electron thermal transport. However, the reader is cautioned that the model used here is quite simplified and is purely electrostatic. More generally, this analysis suggests a general method for tackling the problem of pattern selection

and anisotropy of secondary structures. It also identifies a possible cutoff scale for the zonal-flow spectrum which enters via strongly nonlinear dynamics.

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